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ABSTRACT

A program designed for students who enter the teacher education program with a strong academic background in mathematics is described. However, there is a significant gap between academic course completion and depth and breadth of understanding of mathematics. Teachers' practices reflect their view of mathematics. Therefore, students in the one-semester program are led to reflect on and question their conceptions. A five-phase approach is described, involving: (1) confrontation and reflection; (2) exploration and presentation of a model of mathematics; (3) discussion and problem solving related to the model; (4) modeling of instruction based on the model by the college instructor; and (5) student teachers' incorporation of strategies based on the model. Short-term effects of this program have been documented, but only informal, incidental feedback is available on the long-term effects. Nineteen references are included. (MNS)

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A Secondary Preservice Program's Emphasis on the Nature of Mathematics

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A Secondary Preservice Program's Emphasis on the Nature of Mathematics

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There are some contemporary lay people who believe that a strong academic background in mathematics is sufficient to prepare teachers who will communicate the dynamic nature of the discipline. However, those who have studied both the theory and practice of teaching an academic discipline are aware that there is a significant gap between academic course completion and depth and breadth of understanding of mathematics. Smith (1971), long before research on teachers' conceptions of subject matter became popular, differentiated between knowledge of an academic discipline and knowledge about the discipline. Yet, this difference, though important, still falls short of the need to explore and expand teachers' conceptions of mathematics itself. This is the purpose of the preservice program emphasis to be outlined in this paper.

Significance In the long run a teacher whose teaching practices reflect a view of mathematics as a static discipline or one consisting of disconnected bits probably has effects on the conceptions of his or her students. Colleagues of mine, McDonald and Kouba (1986) have been studying students' conceptions about mathematics. They have asked students, elementary and junior high, to judge situations as to whether the people were doing or not doing mathematics and to give a reason why they made the particular judgment. In an informal diagnostic assignment given to teachers in a graduate class, I have asked that they have students write anonymous descriptions of "What They View as Mathematics." Here is a summary of some of the conceptions about mathematics that students expressed, indirectly or directly, both in the McDonald & Kouba study and in the informal investigation carried out by teachers in the graduate class.

1.Mathematics is a collection of illogical rules.
2.Mathematics is a body of "TRUTHs" , holy to some, self-evident to nobody but the teacher.
3.Mathematics has no relevance to my real world.
4.Mathematics, especially geometry, must be memorized. It cannot be figured out.
5.Mathematicians are people who memorize easily.

6.Mathematicians are people who play games with silly rules.

Buerk (1982) obtained similar, but more articulate statements when she asked able college students to keep journals on their feelings about mathematics.

Notice the chain of attributes implied by these findings. Teacher knowledge --> Teacher conceptions--> Students' conceptions --> Students' learnings. There is some evidence that the theoretical chain I've pictured has links to reality. (See for example, Romberg and Carpenter, 1987). There is also some recent research on teacher's judgments and more particularly, on teacher's conceptions about mathematics (Thompson,1985; Fennema, Carpenter, & Peterson, 1986). There have also been recent calls for more attention to preservice teachers' conceptions about mathematics (Cooney, 1988) in order to implement the curricular reforms outlined in documents, such as the Curriculum and Evaluation Standards for School Mathematics (1987).

The Program Emphasis

The program emphasis outlined in this paper has been in existence since 1973. The preservice teachers enter the program as undergraduate or graduate students with a substantial academic background in mathematics -- equal to and often, greater than, that of the non-teaching mathematics major. The period of time in which these preservice teachers are in this phase of the program is just one semester, about 17 weeks. Students with this kind of background generally assume that their understanding of mathematics is sufficient for the task of teaching secondary school mathematics and can be resistant to exploratory work in the discipline. Thus, it is necessary to lead them to reflect on and question their conceptions. We have developed a five-phase approach that leads the students from reflecting on their own conceptions to practicing relevant strategies in the classroom with their own students. The five phases are:

1. Confrontation and Reflection
2. Exploration and Presentation of a Model of Mathematics
3. Discussion and Problem Solving Related to the Model
4. Modeling of Instruction based on the Model by the College Instructor
5. Student Teacher's Incorporation of Strategies based on the Model

Confrontation and reflection Early in the semester, we use a confrontation tactic to both help these college seniors and graduate students

identify and confront their own conceptions of mathematics, as well as to give us some diagnostic data. Toward the end of a class session, we present the following challenge.

"Is Mathematics Invented or Discovered? You must "vote" on one or the other of these choices. The meaning of *invented* we are using is that associated with people, such as Eli Whitney, who *invented* the cotton gin. Thus, *invented* is the construction or design of something not already in existence. The meaning of *discovered* we are using is that associated with people, such as Columbus, who is said to have *discovered* America. Thus, *discovered* is the finding of something that has been in existence. Both processes can be the result of creative thinking on the part of the doers."

In the more than 20 semesters we have used this strategy, most students vote for *Mathematics being discovered*. Regardless of the outcome, we next question students of either persuasion with regard to specifics that might support their decision. We insist that they give specific examples from mathematics or the history of mathematics. We offer counterexamples or raise questions. At the end of the class session, the students are assigned some reading on the subject (Farrell & Farmer, 1988). (See also Sawyer, 1964; Kline, 1972; Wilder, 1968). They are also encouraged to continue the discussion outside of class and to reflect on the connection between the two views of mathematics and the way mathematics might be communicated to secondary school students.

Exploration and presentation of a model of mathematics A followup assignment requires that students choose one of the courses they are going to teach and write a list of processes and products of mathematics that should be part of that course. In the next class session, the various lists are compared. After some small group and whole class interaction, the students begin to realize that there are some general processes, such as generalizing, and some quite specific ones, such as graphing functions. The students are assigned further reading on processes and products used by mathematicians in aspects of the history of mathematics, as well as those used by teachers in sample excerpts from lessons in secondary school mathematics (See Farrell & Farmer, 1988, Chapter 4). Throughout the reading and the followup class question-answer session, important distinctions between various products of mathematics (Figure 1) are raised.

Defined versus Undefined Concepts
Therorem versus Postulate
Axiom, versus Assumption versus Postulate
Nature of Mathematical Concepts versus those of Science
Physical Models versus Mathematical Models
Proof versus Verification

Figure 1 Distinctions between Important Products of Mathematics

Through the illustrative examples in the readings, students are introduced to important processes of mathematics (Figure 2) -- some of which are more likely to be used in the collection of data.

Defining	**Data reducing**
Assuming	**Data collecting**
Deducing	**Graphing**
Classifying	**Interpolating**
Symbolizing	**Extrapolating**
Generalizing	**Inferring**
Modeling	**Inducing**

Figure 2: Selected Processes of Mathematics

As a result of the readings and the class interaction, the students now have a concrete base to better understand the Model of Mathematics (Figure 3) to be presented and discussed in class.

Insert Figure 3 about here

The model is studied and discussed. The students are asked to consider the graphics in the model and the interpretation the designers placed on these graphics and they are encouraged to point out limitations of the model. It has been our experience that the introduction of the model of mathematics must be followed closely by student problem solving and discussion so that the students can relate their strategies and results to some of the processes and products listed in the model.

Discussion and problem solving based on the model Once again, the

students are asked to complete some background reading. They are also asked to engage in problem solving of the type that relates to secondary school mathematics and to reflect on their thinking strategies. Some of the activities that they have engaged in include: the muscle fatigue laboratory, study and discussion of the telephone problem, and paper folding to obtain the conics. The original source for the muscle fatigue lesson is the School Mathematics Study Group text, Mathematics and living things (1965). It was designed for mathematics classrooms and demonstrates a nice use of the interface of mathematics and science. Data are collected, reduced and studied. Relevant real world questions flow from the graphs of the data although students can readily see that the graphs become a mathematical model with some limitations. The telephone problem can be found in Bell (1972). The students are asked to study a phone company map of the United States, which can be found in any phone book. They, then, are asked to conjecture the pattern, if any, for assigning area codes to various geographical areas of the United States. The novelty of the question and its unquestionable relevance to the real world are intriguing. Finally, the paper folding laboratory can be found in many sources (See e.g. Farrell & Farmer & 1988, Chapter 3) . In this case, the simple activity is presented without telling students what the final result will be.

Throughout all of these sample activities, the students have an opportunity to engage in mathematics in a non-standard way. Most important to the issue of helping them reflect on the nature of mathematics is the class followup that goes beyond solutions to an examination of processes and products in which they engaged and the connections to the model of mathematics.

Modeling of instruction based on the model Throughout the preceding phase of the semester, the college instructor is modeling instruction based on the model. In order to make this explicit to the preservice students, they are introduced to competencies that they are expected to exhibit in their own classrooms -- competencies that are derived from the model of mathematics (Figure 4).

Insert Figure 4 about here

Each week, the students work with the instructor to identify specific activities, strategies, lessons, assignments in the college class that are illustrate specific competencies in Figure 4. They also study the connections between the competencies and the model of mathematics. (There

are other equally important competencies related to the cognitive development of the students and the view that students construct their own knowledge. These overlap the mathematical competencies in Figure 4 in important ways. For further information on these competencies, see Farrell ,Farmer, and Clark [1976] and Farmer, Farrell, and Clark [1977 and 1978].)

Student teacher's incorporation of strategies based on the model Our students spend one day a week for the first four weeks of the professional semester in the school in which they will be student teaching. From the fifth to the fifteenth week, they spend all day, every day, at the school, gradually assuming teaching responsibility for the classes of their cooperating teacher. They are expected to model the competencies studied in the first four weeks in their own classrooms. Careful observation by the college instructor who taught them and understands the theoretical rationale of the program is a key ingredient to having the model realized in the classroom of these novice teachers. Throughout the clinical experience, they are given feedback on what they are doing with respect to the desired teaching competencies with midway assessment of the extent to which they are progressing. One example of classroom observation data that might show progress or lack of progress. is given in Figure 5.

Figure 5 about here

Before being allowed to assume a second class, the student teacher must effectively meet at least some of the competencies listed in Figure 4. By the end of the program, successful students demonstrate competence in manyof the competencies, show progress in others, and may still need to work at others. In the exit interview with these student teachers, the ongoing nature of learning to be a teacher is emphasized.

Summary The short-term effects of this kind of program have been documented in three research studies (Farrell, Farmer, & Clark, 1976; Farmer, Farrell, & Clark, 1977 and 1978) and one dissertation (Stone, 1976). Thus far, we have only informal and incidental feedback on the long-term effects of this kind of emphasis. One kind of feedback comes from area department heads and principals who have hired our students and who report on their effectiveness. A second kind of feedback comes from our own observations of the classes of a sample of these graduates who agreed to be cooperating teachers for a new generation of student teachers. The feedback from both of these sources has been positive, but we readily admit to the selective nature of both sources of feedback. A dissertation in progress

(Feldt) is designed to pay specific attention to teachers' conceptions of the nature of mathematics, the sources of those conceptions, and the extent to which they are reflected in classroom practice. We await these results with interest. We believe, with Alan Bishop (1972) that "... the best way to improve teaching is to help the teacher to think about what he (she) is doing to help him (her) to improve his (her) private theories. For so much depends on how you think about what you are doing when you are teaching."

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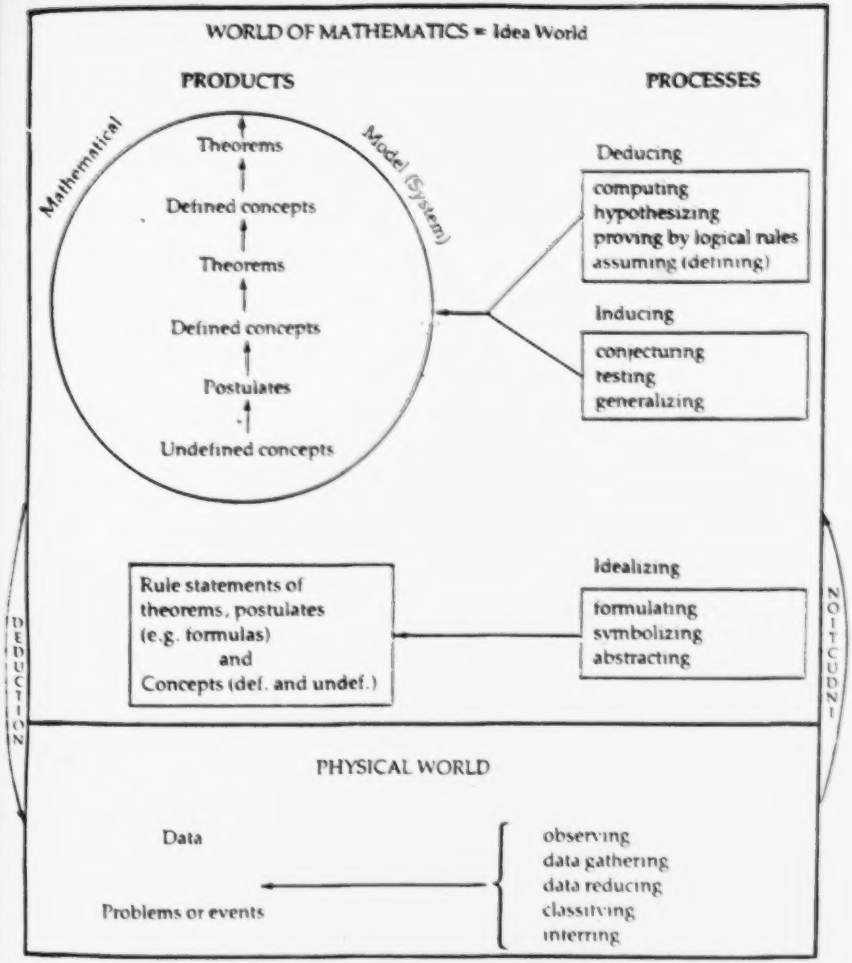

Figure 3: Model of Mathematics

See Farrell and Farmer, 1988.

3. Clarified the Processes and Ideas of Mathematics

3.1 Provided students with experiences in obtaining mathematical generalizations (patterns, rules, or principles)

3.2 Provided students with experiences in using problem-solving strategies (e.g. testing and modifying conjectures, restructuring the problem)

3.3 Distinguished between generalizations obtained from data and those obtained by deduction from assumptions

3.4 Provided a rationale for the terms, definitions, and algorithms of mathematics

3.5 Provided illustrations to show that mathematics is an invention of the human mind and an activity of human beings

3.6 Provided illustrations to show the distinctions between the ideal world of mathematical models (thought models) and the physical world in which the models are applied

3.7 Translated mathematical concepts and principles (rules) into valid forms (symbolic, schematic, or spatial)

3.8 Called attention to content inaccuracies which appeared in verbal interaction, instructional materials, and student papers

3.9 Used correct and consistent mathematical language and symbolism

3.10 Demonstrated sufficient depth and breadth of understanding in mathematics to respond to unplanned activities, to "go beyond" text materials, and to spiral the teaching of concepts

3.11 Related mathematics to other school subjects, especially science

3.12 Provided examples of the application of mathematics to areas outside of school subjects

Figure 4: Competencies in Mathematics

See Farrell, Farmer, & Clark, 1976.

Competency 3.4 Provided a rationale for the terms, definitions, and algorithms of mathematics

Indicators of (T) competency	Indicators of (T) non-competency
Viewing of Teacher	
T gives history, etymology of new terms and symbols T tells Ss the connection between topics, the reason(s) why (e.g., factoring techniques are being taught). T correctly indicates the "why" of certain conventions. T uses counting and measuring examples before a new formula is developed and points out the usefulness of the formula.	T gives no explanation for a term's use in two different ways (e.g., congruent). T's response to certain principles of mathematics is: "That's an agreement made by mathematicians; Learn It."

Figure 5: Sample Classroom Observation Data